


# From theory to Gravitational Waves for cosmological phase transitions

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Chengdu, 04/2026

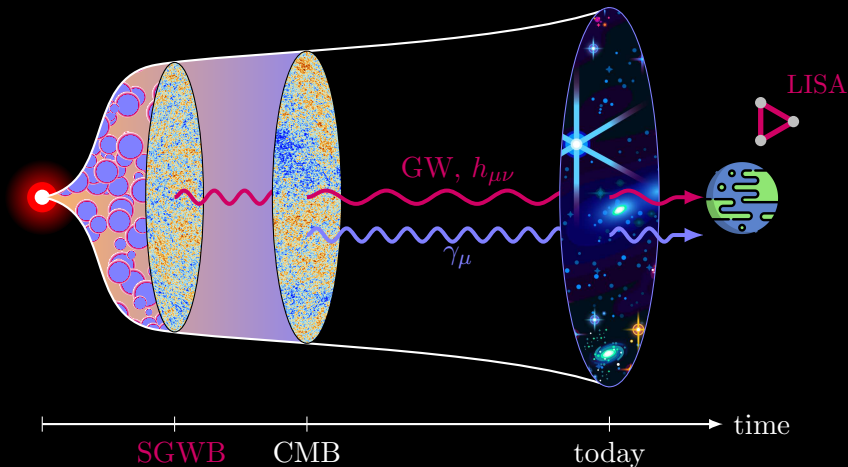
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 at weak coupling.

Supported by the SNSF under grant PZ00P2-215997.

# Gravitational Waves (GWs) can be

sourced by early-universe **phase transitions** caused by new physics.

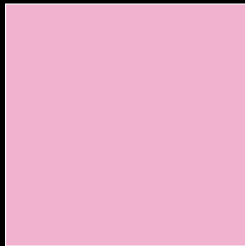
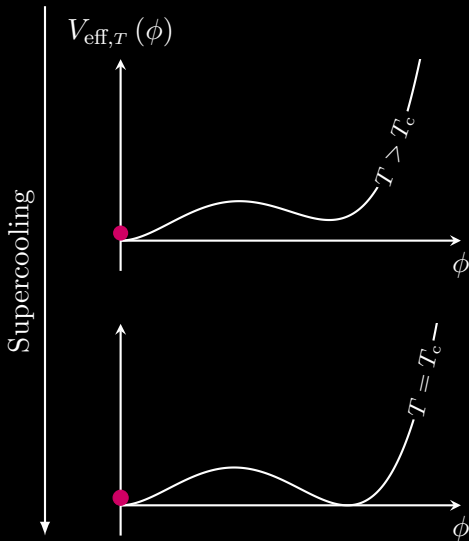


Cosmic Microwave Background (CMB)

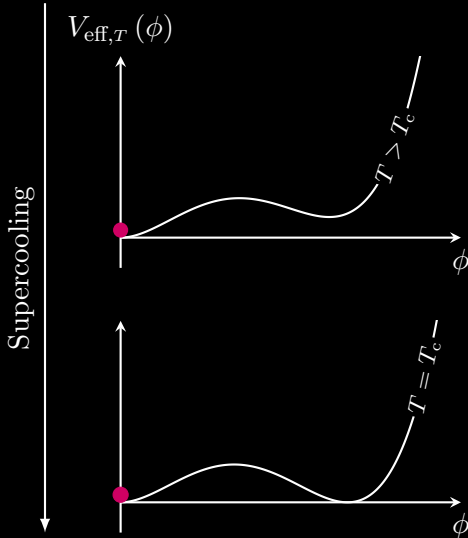
Stochastic GW Background (SGWB)

Laser Interferometer Space Antenna (LISA)

# Cosmological first order phase transition

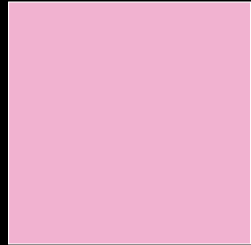


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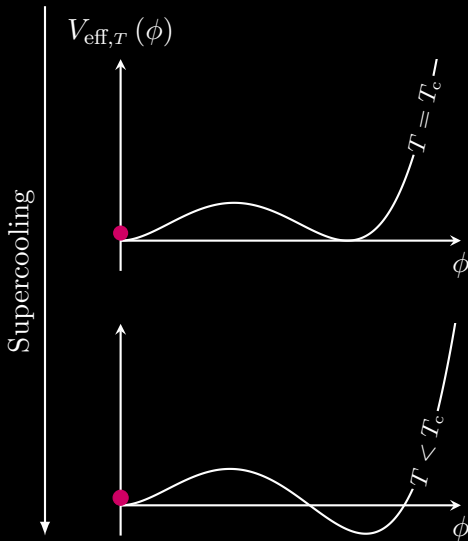


UV [thermal] corrections modify the zero- $T$  potential and restore symmetry.

$$V_{\text{eff},T} \supset \text{[diagrams]} \dots$$

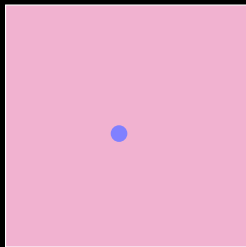


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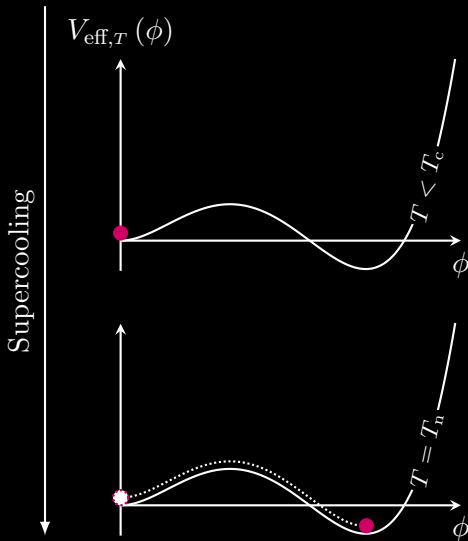


When the universe cools down, the field transitions.

If a barrier separates the phases, the transition is first order.

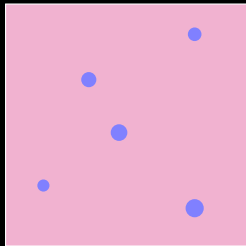


# Cosmological first order phase transition



The phase transition proceeds via nucleation of bubbles.

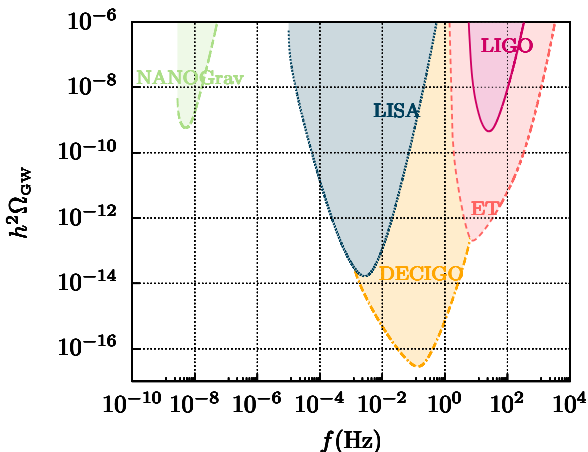
Part of the released energy can be converted into gravitational waves:  $\square h_{ij} = 4\pi G T_{ij}$



# The gravitational wave signal is stochastic

and the sum of independent events (interactions between the bubbles).

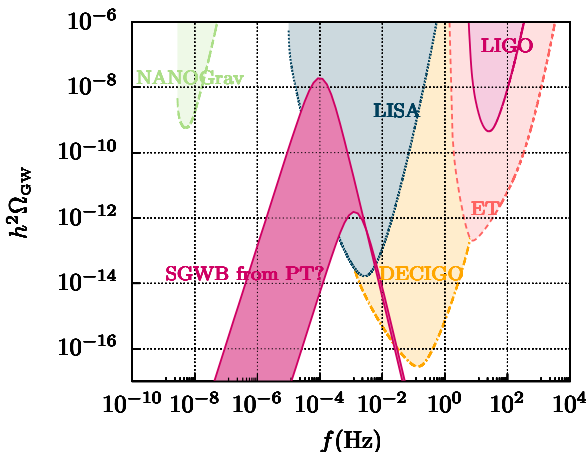
The spectral parameters (amplitude, peak frequency, powers in the IR/UV) contain information about the source.



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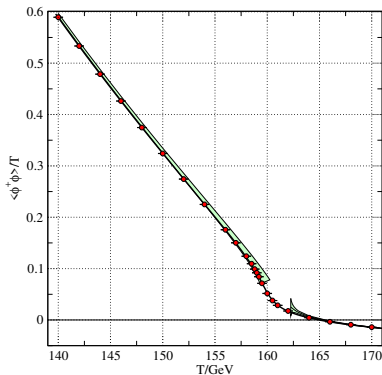
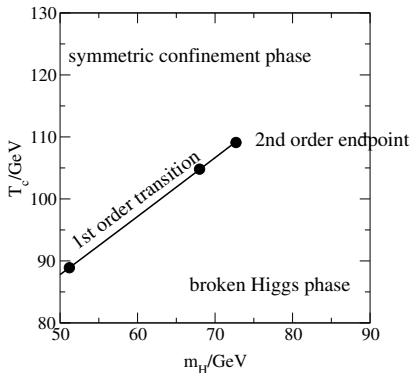
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# GWs from a cosmological phase transition: New physics?

In Standard Model, EWSB occurs via a smooth crossover<sup>1</sup> but possible that it is first-order in Beyond the Standard Model (BSM) extensions.



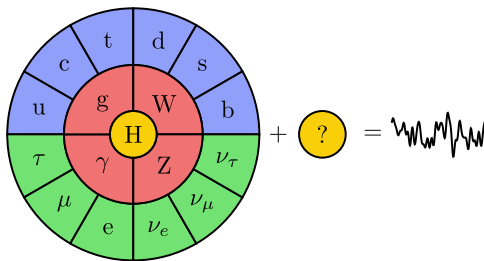
<sup>1</sup> Kajantie, Laine, Rummukainen, Shaposhnikov, *Is there a hot electroweak phase transition at  $m_H \gtrsim m_W$ ?*, Phys. Rev. Lett. **77** (1996) 2887 [hep-ph/9605288], D'Onofrio, Rummukainen, *Standard model cross-over on the lattice*, Phys. Rev. D **93** (2016) 025003 [1508.07161]

# GWs from a cosmological phase transition: New physics?

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Study **BSM physics** near EW scale in context of phase transitions:

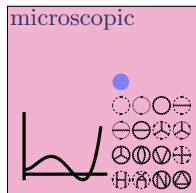
- ▷ Light fields strongly coupled to Higgs
- ▷ Collider targets. BSM testing pipeline: Collider phenomenology



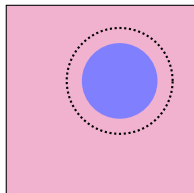
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# Connecting micro- and macrophysics

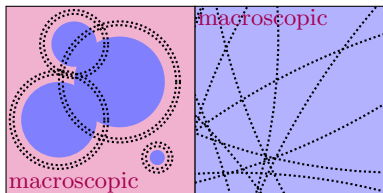
Large dynamic range to resolve  $\Omega_{\text{GW}}$  at both low and high frequency.  
Reliable GW predictions require control at **each** scale.



Bubble nucleation  
thermodynamics



growth  
wall-fluid system

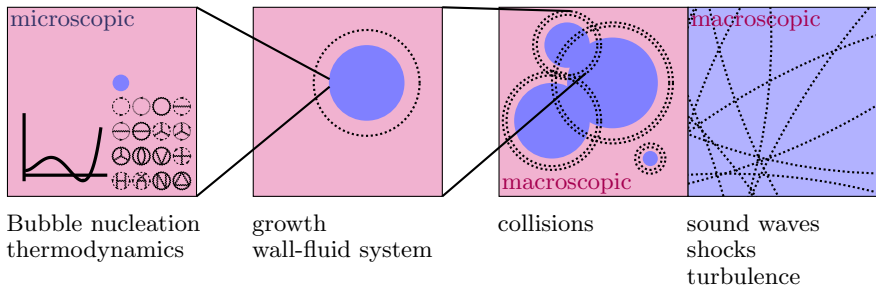


collisions

sound waves  
shocks  
turbulence

# Connecting micro- and macrophysics

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Strategy:

**Effective field theory (EFT)** to connect **micro-** and **macrophysics**.

# Fingerprinting the GW spectrum via microphysics

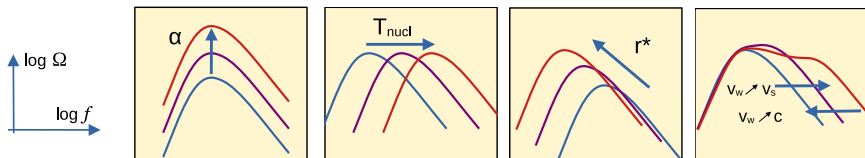
using a few equilibrium parameters:

$T_\star$  reference temperature of the transition ( $T_\star = T_{\text{nucl}}, T_p$ ),

$\alpha$  phase transition strength,

$\beta/H$  inverse duration of the transition (also  $r_\star$ ),

$c_s$  speed of sound in the plasma



$$h^2 \Omega_{\text{GW}} = h^2 \Omega_{\text{GW}} \left( T_\star, \alpha, \frac{\beta}{H}, c_s, \dots \right)$$

# How robust are theoretical predictions?

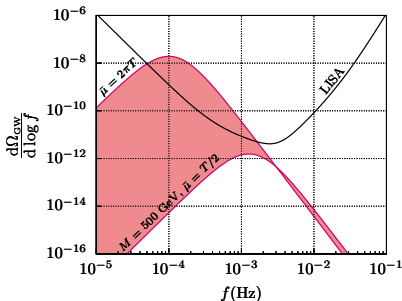
Uncertainties of  $\mathcal{O}(10^4)$  even for purely perturbative regimes<sup>2</sup>.

$\Omega_{\text{GW}}$  depends strongly on **microphysics** in **macroscopic** simulation fits:

$$h^2\Omega_{\text{GW}} = h^2\Omega_{\text{GW}}\left(T_\star, \alpha, \frac{\beta}{H}, c_s, \dots\right).$$

Vary RG scale  $\bar{\mu}$  in SM extensions:

- ▷ SMEFT:<sup>3</sup> SM +  $\frac{1}{M^2}(\phi^\dagger\phi)^3$
- ▷ xSM:<sup>4</sup> SM + singlet
- ▷ BSM models in general



<sup>2</sup> Croon, Gould, Schicho, Tenkanen, White, *Theoretical uncertainties for cosmological first-order phase transitions*, JHEP **04** (2021) 055 [2009.10080]

<sup>3</sup> Chala, Fiore, Gil, *Hot news on the phase-structure of the SMEFT*, [2507.16905]

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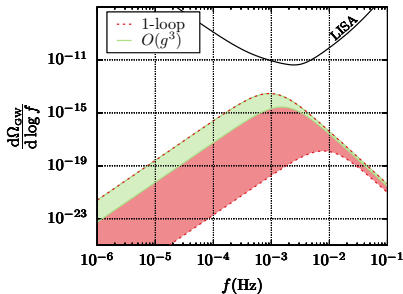
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# Perturbative phase transitions need scale hierarchies

for quantum effects  $\Delta V_{\text{fluct}}$  to influence the tree-level potential

$$V_{\text{eff}} = V_{\text{tree}} + \Delta V_{\text{fluct}} .$$

Relevant operators ( $\sigma > 0$ ) in the IR get large UV contributions<sup>5</sup> and

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim g^2 \left[ \frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}} \right]^\sigma \stackrel{!}{\sim} 1$$

(i) strong coupling

$$g^2 \gtrsim 1$$

How to reorganize perturbation theory for large  $\Delta V_{\text{fluct}}$ ?

---

<sup>5</sup>Consider e.g. hierarchy:  $M_\chi \gg m_\Phi$ ,  $(\Delta m_\Phi^2) \Phi^\dagger \Phi = \text{---} \sim g^2 M_\chi^2 \Phi^\dagger \Phi$ ,  $(\Delta m_\Phi^2)/m_\Phi^2 = g^2 [M_\chi/m_\Phi]^2$ .

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(ii) scale hierarchy

$$\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}} \sim g^{-\frac{2}{\sigma}} \gg 1$$

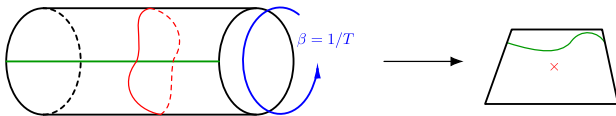
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# A robust EFT framework at high temperatures: 3d EFT<sup>6</sup>

Thermodynamics  $\mathcal{Z} = \text{Tr} e^{-\beta\mathcal{H}}$  formulated in  $\mathbb{R}^3 \times S^1_\beta$ .

$$\underbrace{\int_{\mathbb{R}^3 \times S^1_\beta} \mathcal{L}}_{\text{bosons + fermions}} \rightarrow \underbrace{\int_{\mathbb{R}^3} \mathcal{L}^{\text{eff}}}_{\text{bosons only}} .$$



In 3d EFT, modified canonical mass dimensions render marginal operators relevant,  $[A_0^a] = [g_{3,abc}] = 1/2$ : **superrenormalizable** theory.

$$\int_{\mathbb{R}^3} \mathcal{L}^{\text{eff}} \supset \left[ \frac{1}{2} \partial_i A_0^a \partial_i A_0^b - g_{3,abc} A_i^a A_0^b \partial_i A_0^c + \frac{m_{3,ab}^2}{2} A_0^a A_0^b \right] .$$

<sup>6</sup> Braaten, Nieto, *Effective field theory approach to high temperature thermodynamics*, Phys. Rev. D **51** (1995) 6990 [hep-ph/9501375],

# Powercounting the SM-like 3d EFT (SU(2)+Higgs)

Describes the thermodynamics<sup>7</sup> of several parent 4d theories:

$$\mathcal{L}_{3d}^{\text{softer}} = \frac{1}{4} F_{ij}^a F_{ij}^a + (D_i \Phi)^\dagger (D_i \Phi) + V(\Phi),$$
$$V(\Phi) = m_3^2 \Phi^\dagger \Phi + \lambda_3 (\Phi^\dagger \Phi)^2.$$

If  $m_{A_i} \sim g_3 \phi \gg m_3$ , integrating out vector boson introduces LO barrier

$$V_{\text{LO}}(\Phi) = \bullet + \text{blob}.$$

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<sup>7</sup> Kajantie, Laine, Rummukainen, Shaposhnikov, *Generic rules for high temperature dimensional reduction and their application to the standard model*, Nucl. Phys. B **458** (1996) 90 [hep-ph/9508379]

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$$V_{\text{LO}}(\Phi) = m_3^2 \Phi^\dagger \Phi + \lambda_3 (\Phi^\dagger \Phi)^2 - \frac{g_3^3}{2\pi} \left( \frac{\Phi^\dagger \Phi}{2} \right)^{3/2}.$$

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If  $m_{A_i} \sim g_3 \phi \gg m_3$ , integrating out vector boson introduces LO barrier

$$V_{\text{LO}}(\Phi) \rightarrow y \Phi^\dagger \Phi + x (\Phi^\dagger \Phi)^2 - \frac{1}{2\pi} \left( \frac{\Phi^\dagger \Phi}{2} \right)^{3/2}.$$

Since  $x \sim \frac{m_3^2}{m_{A_i}^2} \ll 1$ , and at the phase transition  $y \sim 1/x$ , we strictly<sup>8</sup>  $x$ -pand the perturbative series using 3d EFT dimensionless couplings

$$x \equiv \frac{\lambda_3}{g_3^2}, \quad y \equiv \frac{m_3^2}{g_3^4}.$$

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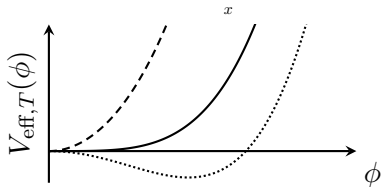
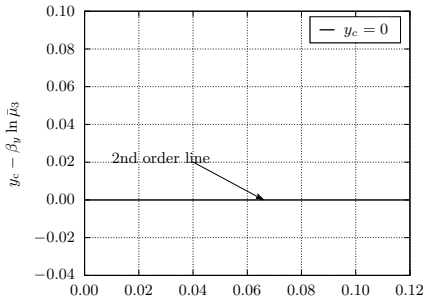
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# The electroweak phase diagram

One-loop

$$V_{\text{eff},T} = V_0 + \text{[loop diagram]} \\ = \frac{1}{2} m_3^2(T) \phi^2 + \frac{1}{4} \lambda_3(T) \phi^4$$



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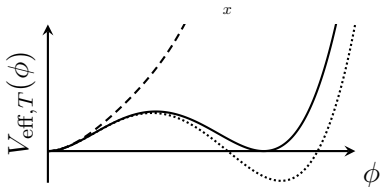
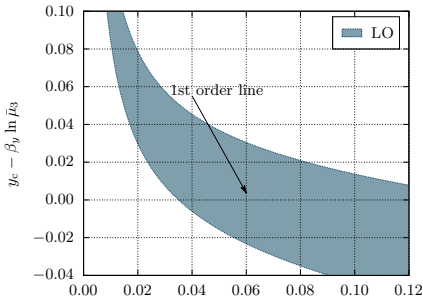
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Leading-order resummations

$$\begin{aligned} V_{\text{eff},T} &= V_0 + \text{---}\text{---}\text{---} \\ &= \frac{1}{2}m_3^2\phi^2 + \frac{1}{4}\lambda_3\phi^4 - \frac{g_3^3}{16\pi}\phi^3 \end{aligned}$$

An EFT  $x$ -pansion<sup>9</sup> in

$$x = \lambda_3/g_3^2.$$



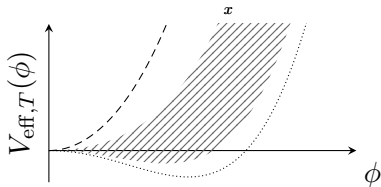
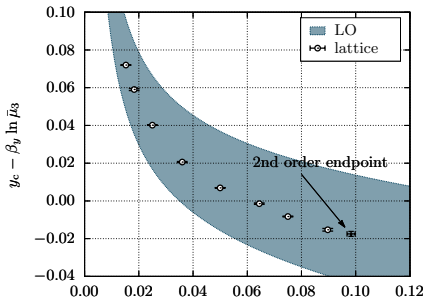
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Infrared breakdown:

$$V_{\text{eff}} = ?$$

All higher loops  $\subset$  N<sup>5</sup>LO. Resolve with lattice Monte-Carlo.<sup>10</sup>



<sup>10</sup> Farakos, Kajantie, Rummukainen, Shaposhnikov, *3-d physics and the electroweak phase transition: A Framework for lattice Monte Carlo analysis*, Nucl. Phys. B **442** (1995) 317 [hep-lat/9412091], Kajantie, Laine, Rummukainen, Shaposhnikov, *The Electroweak phase transition: A Nonperturbative analysis*, Nucl. Phys. B **466** (1996) 189 [hep-lat/9510020], Gould, Güyer, Rummukainen, *First-order electroweak phase transitions: A nonperturbative update*, Phys. Rev. D **106** (2022) 114507 [2205.07238]

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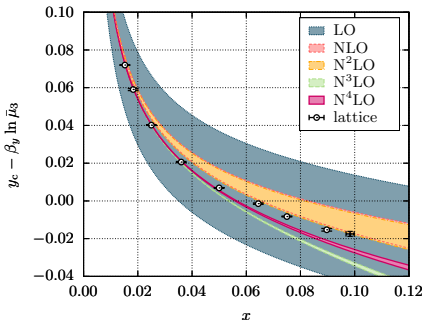
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Remarkable agreement at

- ▷ NLO<sup>11</sup>
- ▷ N<sup>2</sup>LO<sup>12</sup>
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<sup>11</sup> Arnold, Espinosa, *Effective potential and first-order phase transitions: Beyond leading order*, Phys. Rev. D **47** (1993) 3546 [9212235]

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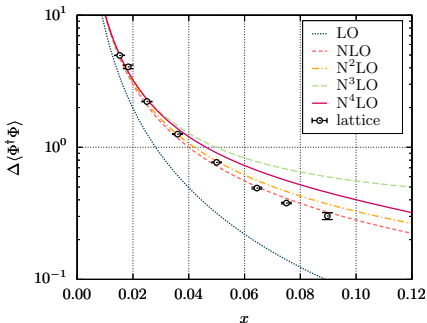
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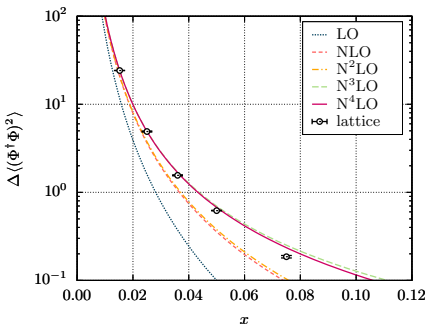
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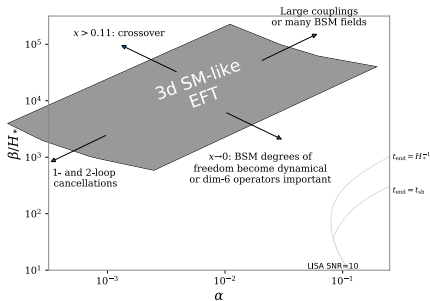
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# Beyond the SM and evading the rhombus of misery

SM-like EFT  $\beta/H[\alpha]$  rhombus has no prospect for large SNR.<sup>14</sup>



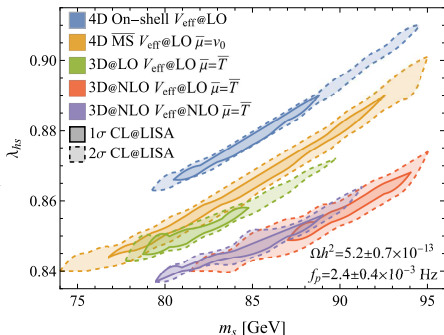
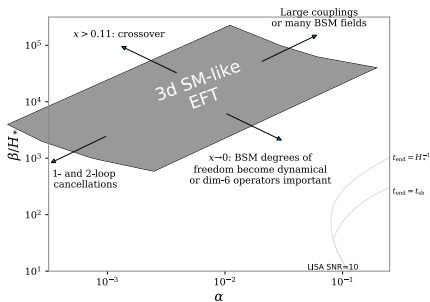
<sup>14</sup> Gould, Kozaczuk, Niemi, Ramsey-Musolf, Tenkanen, Weir, *Nonperturbative analysis of the gravitational waves from a first-order electroweak phase transition*, Phys. Rev. D **100** (2019) 115024 [1903.11604]

<sup>15</sup> Lewicki, Merchand, Sagunski, Schicho, Schmitt, *Impact of theoretical uncertainties on model parameter reconstruction from GW signals sourced by cosmological phase transitions*, Phys. Rev. D **110** (2024) 023538 [2403.03769], Niemi, Schicho, Tenkanen, *Singlet-assisted electroweak phase transition at two loops*, Phys. Rev. D **103** (2021) 115035 [2103.07467]; See talk by **A. Yang** on **Sun 15:55**

# Beyond the SM and evading the rhombus of misery

SM-like EFT  $\beta/H[\alpha]$  rhombus has no prospect for large SNR.<sup>14</sup>

Parameter reconstruction only robust after (2+)-loop resummation.<sup>15</sup>



<sup>14</sup> Gould, Kozaczuk, Niemi, Ramsey-Musolf, Tenkanen, Weir, *Nonperturbative analysis of the gravitational waves from a first-order electroweak phase transition*, Phys. Rev. D **100** (2019) 115024 [1903.11604]

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# Dimension-six operators in $U(1) + \text{Higgs}^{16}$

WHAT IF WE TRIED  
MORE LOOPS ?



So far **truncated operators** at high  $T$  at dimension 4:

$$S_{\text{soft}}^{3\text{d}} = \int_{\mathbf{x}} \left\{ \mathcal{L}_{\text{soft}}^{3\text{d}} + \sum_{n \geq 5} \frac{\mathcal{O}_n}{(\pi T)^n} \right\},$$

$$S_{\text{softer}}^{3\text{d}} = \int_{\mathbf{x}} \left\{ \mathcal{L}_{\text{softer}}^{3\text{d}} + \sum_{n \geq 5} \frac{\mathcal{O}_n}{(m_{\text{D}})^n} \right\}.$$

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<sup>16</sup> Bernardo, Klose, Schicho, Tenkanen, *Higher-dimensional operators at finite temperature affect gravitational-wave predictions*, JHEP **08** (2025) 109 [2503.18904], Bernardo, Chala, Gil, Schicho, *Hard thermal contributions to phase transition observables at NNLO*, [2602.06962]

# Dimension-six operators in $U(1) + \text{Higgs}$ <sup>16</sup>

WHAT IF WE TRIED  
MORE LOOPS AT ADEQUATE  
ORDER OF  
HIGHER DIMENSIONAL OPERATORS ?



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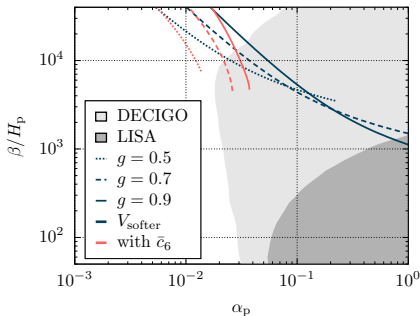
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# Gravitational wave prospects are compromised

in the LISA-relevant regime due to **breakdown of high- $T$  expansion**.  
**Softer scale** yields large differences for strongest transitions.



$$S_{\text{softer}}^{3\text{d}} = \int_{\mathbf{x}} \left\{ \mathcal{L}_{\text{softer}}^{3\text{d}} + \sum_{n \geq 5} \frac{\mathcal{O}_n}{(m_{\text{D}})^n} \right\}$$

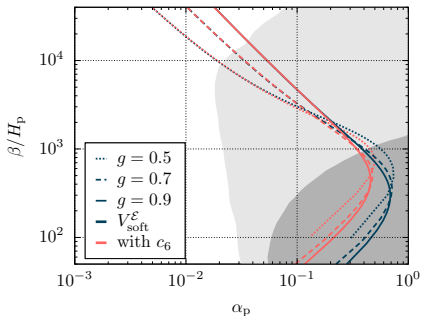
**Limitation:** conventional lattice results of (dim-4) super-renormalizable 3d EFT do **not describe hard/soft-scale** driven transitions.<sup>17</sup>

Need **4d full theory lattice** simulations for a reliable description?

<sup>17</sup> Laine, *Comparison of 4-D and 3-D lattice results for the electroweak phase transition*, Phys. Lett. B **385** (1996) 249 [hep-lat/9604011]

# Gravitational wave prospects are compromised

in the LISA-relevant regime due to **breakdown of high- $T$  expansion**.  
**Soft scale enhances** phase-transition strength,  $\alpha(T)$ .



$$S_{\text{soft}}^{3\text{d}} = \int_{\mathbf{x}} \left\{ \mathcal{L}_{\text{soft}}^{3\text{d}} + \sum_{n \geq 5} \frac{\mathcal{O}_n}{(\pi T)^n} \right\}$$

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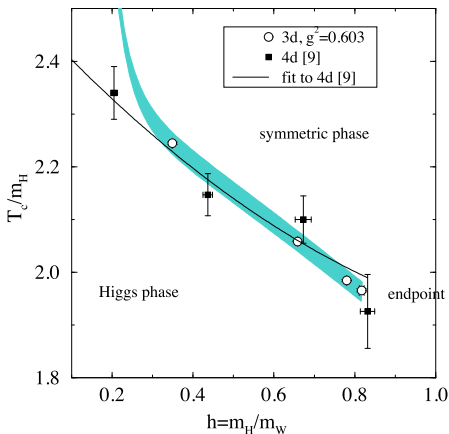
# New lattice benchmarks<sup>18</sup> for strong transition tests

Breakdown is no surprise and was already known since 90s.<sup>19</sup>

Need broad- $T$  frameworks.

Not needed for weak SM-like transitions.

Relevant for:  
strong **BSM** transitions.



<sup>18</sup>Annala, Hallfors, Rummukainen, Seppä, Schicho, Tenkanen, in progress

<sup>19</sup> Laine, *The Renormalized gauge coupling and nonperturbative tests of dimensional reduction*, JHEP **06** (1999) 020 [hep-ph/9903513], Csikor, Fodor, Heitger, *Endpoint of the hot electroweak phase transition*, Phys. Rev. Lett. **82** (1999) 21 [hep-ph/9809291]

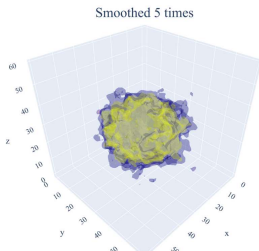
# Beyond equilibrium: Bubble nucleation rate

determines the nucleation temperature ( $T_n$ ) and duration of the phase transition  $(\beta/H)^{20}$

$$\Gamma = \Gamma_{\text{dyn}} \times \Gamma_{\text{stat}} \sim A e^{-B},$$

*B* Internal energy of critical bubble. Solve one non-linear ODE

$$\frac{d^2\phi_b}{dr^2} + \frac{2}{r} \frac{d\phi_b}{dr} - V'(\phi_b) = 0$$



<sup>20</sup> Langer, Turski, *Hydrodynamic model of the condensation of a vapor near its critical point*, Physical Review A **8** (1973) 3230; figure for critical bubble in SU(2)-Higgs model by Gould, Güyer, Rummukainen, *First-order electroweak phase transitions: A nonperturbative update*, Phys. Rev. D **106** (2022) 114507 [2205.07238]

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$$\frac{d^2 \phi_b}{dr^2} + \frac{2}{r} \frac{d\phi_b}{dr} - V'(\phi_b) = 0$$

*A* log entropy of fluctuations about bubble. Infinite linear ODEs

$$\left[ \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{l(l+1)}{r^2} + V''(\phi_b) \right] \psi_l^b(r) = 0$$

Need both *A* and *B* for **reliable** rate computation. EFT for  $\Gamma_{\text{stat}}$ <sup>21</sup>

$$\Gamma_{\text{stat}} \sim \left[ m_{\text{eff}}^4 e^S \right] \times e^{-E_b/T} \sim m_{\text{eff}}^4 e^{-(E_b - ST)/T}.$$

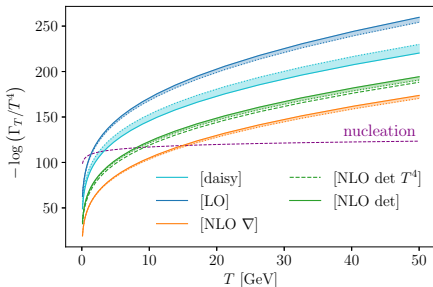
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<sup>20</sup> Langer, Turski, *Hydrodynamic model of the condensation of a vapor near its critical point*, Physical Review A **8** (1973) 3230

<sup>21</sup> Gould, Hirvonen, *Effective field theory approach to thermal bubble nucleation*, Phys. Rev. D **104** (2021) 096015 [2108.04377]

# Determining the Bubble determinant $A^{22}$ matters

for classically conformal theories with strong supercooling.<sup>23</sup>



Nucleation not just energy and entropy  $\rightarrow$  out-of-equilibrium effects<sup>24</sup>

$$\Gamma = \frac{\kappa_{\text{dyn}}}{2\pi} m_{\text{eff}}^3 e^{-(E_b - ST)/T}.$$

<sup>22</sup> Ekstedt, Gould, Hirvonen, *BubbleDet: a Python package to compute functional determinants for bubble nucleation*, JHEP **12** (2023) 056 [2308.15652]

<sup>23</sup> Kierkla, Schicho, Swiezewska, Tenkanen, van de Vis, *Finite-temperature bubble nucleation with shifting scale hierarchies*, JHEP **07** (2025) 153 [2503.13597]; See talk by **A. Salvio** on **Sun 20:35**

<sup>24</sup> Langer, *Statistical theory of the decay of metastable states*, Ann. Phys. (N. Y.) **54** (1969) 258, Hirvonen, *Real-time nucleation and off-equilibrium effects in high-temperature quantum field theories*, Phys. Rev. D **111** (2025) 116020 [2403.07987]

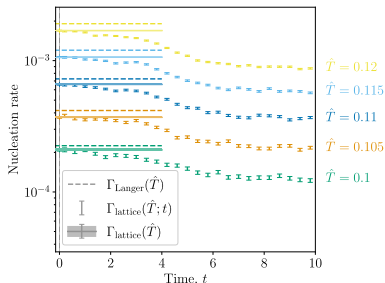
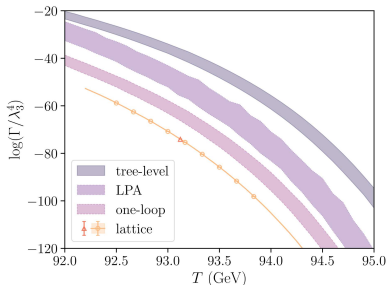
# Agreement between perturbation theory and lattice?

Disagreement<sup>25</sup> because initial state is not thermalized on the lattice?

Recent  $1+1D$  lattice simulations better match perturbation theory.<sup>26</sup>

Agreement up to two-loop order for a thermalized initial state.<sup>27</sup>

The rate is not constant, but **decays over time**.

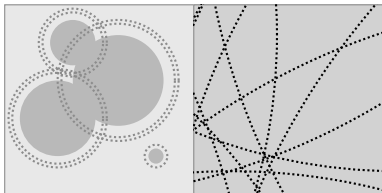
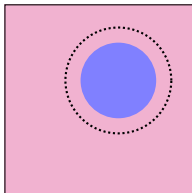
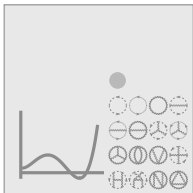


<sup>25</sup> Gould, Kormu, Weir, *Nonperturbative test of nucleation calculations for strong phase transitions*, Phys. Rev. D **111** (2025) L051901 [2404.01876]

<sup>26</sup> Pîrvu, Shkerin, Sibiryakov, *Thermal false vacuum decay in (1+1) dimensions: Evidence for nonequilibrium dynamics*, Int. J. Mod. Phys. A **39** (2024) 2445007 [2408.06411]

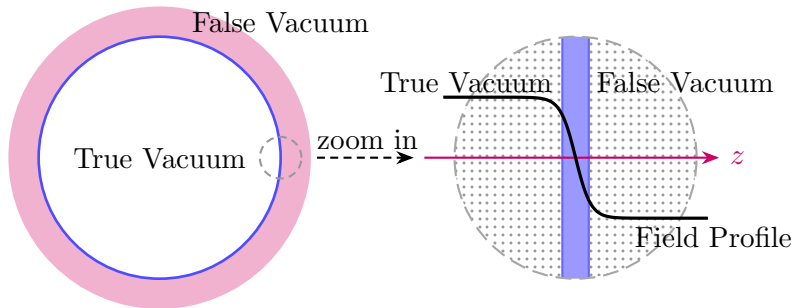
<sup>27</sup> Hirvonen, Gould, *Langer's Nucleation Rate Reproduced on the Lattice*, Phys. Rev. Lett. **136** (2026) 081601 [2505.22732]

## In between scales



# Microscopic description of the bubble wall

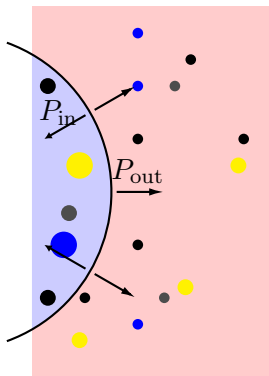
The bubble wall is the region where the field(s) interpolate between the high- and low-temperature vacuum.



# Bubbles typically reach terminal velocity before colliding if

$$|P_{\text{out}}| = |P_{\text{in}}|.$$

- $\Delta P_{\text{out}}$  Vacuum energy release  
(accelerating wall)
- $\Delta P_{\text{in}}$  Particle friction on the wall  
(decelerating wall)
- $\Delta P_{\text{in}}$  Hydrodynamic backreaction  
(decelerating wall)



Iteratively solve coupled **fluid**, **scalar field**, **Boltzmann** equations.<sup>28</sup>

Public software for  $v_w$ :<sup>29</sup> **WA(GO)**.

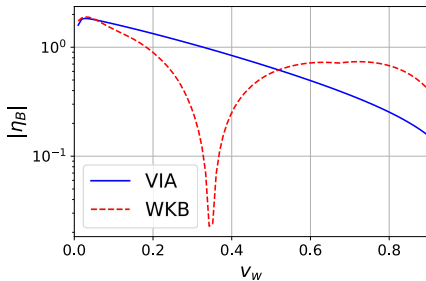
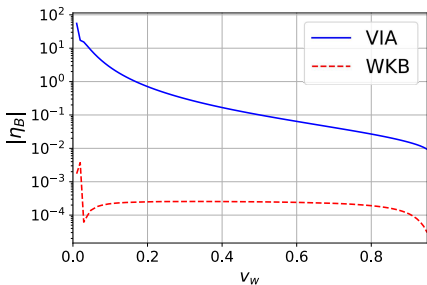
<sup>28</sup> Moore, Prokopec, *How fast can the wall move? A Study of the electroweak phase transition dynamics*, Phys. Rev. D **52** (1995) 7182 [hep-ph/9506475], Laurent, Cline, *First principles determination of bubble wall velocity*, Phys. Rev. D **106** (2022) 023501 [2204.13120],

<sup>29</sup>Schicho, Ekstedt, Gould, Hirvonen et al., *How fast does the WallGo? A package for computing wall velocities in first-order phase transitions*, JHEP **04** (2025) 101 [2411.04970].

# Electroweak baryogenesis depends on the wall velocity<sup>30</sup>

Baryogenesis from CP violation via lepton higher-dimensional Yukawa.

Baryogenesis by  $c$ - $t$  or  $b$ - $s$  flavor mixing.

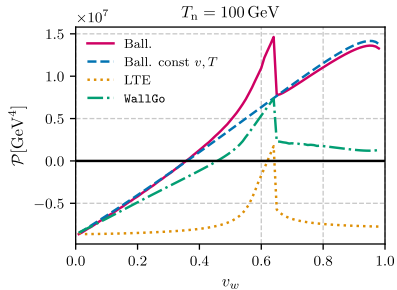
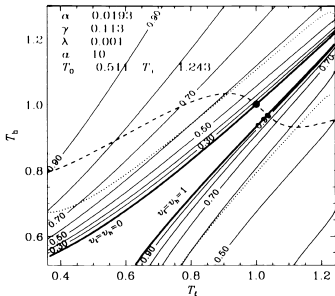


<sup>30</sup> Cline, Laurent, *Electroweak baryogenesis from light fermion sources: A critical study*, Phys. Rev. D **104** (2021) 083507 [2108.04249]; See talk by **M. R. Musolf** on **Thu 09:50**

# Pinching $v_w$ with LTE and ballistic limit

**Upper bound.** Large collision term ( $\mathcal{C}_a \rightarrow \infty$  and  $\delta f^a \rightarrow 0$ ):  
Local thermal equilibrium (LTE).<sup>31,32</sup>

**Lower bound.** Small collision term ( $\mathcal{C}_a \rightarrow 0$ ):  
Ballistic limit.<sup>33</sup> Can solve Boltzmann equation analytically.



<sup>31</sup> Enqvist, Ignatius, Kajantie, Rummukainen, *Nucleation and bubble growth in a first order cosmological electroweak phase transition*, Phys. Rev. D **45** (1992) 3415

<sup>32</sup> Backreaction from hot fluid, and entropy production ( $\Delta_{su} z \geq 0$ ) can still stop the wall c.f. Eriksson, Laine, *Entropy production at electroweak bubble walls from scalar field fluctuations*, JCAP **09** (2025) 027 [2507.07755]

<sup>33</sup> Ai, Laurent, van de Vis, *Bounds on the bubble wall velocity*, JHEP **02** (2025) 119 [2411.13641] at  $m_s = 120 \text{ GeV}$  and  $\lambda_{hs} = \lambda_s = 1$

# Theoretical uncertainties in the wall velocity $\Delta v_w$ <sup>34</sup>

and the driving pressure  $\Delta P$ , are quantified by

$$\Delta v_w \approx \Delta P \left( \frac{dP}{dv_w} \right)^{-1}, \quad \text{using } \text{WA}(\text{GO}).$$

Source of uncertainty		$\Delta v_w/v_w$		
		xSM	IDM	$\mathcal{O}(\Delta P/P)$
Boltzmann	Linearization	0.1%	0.01%	$\mathcal{O}(\delta f_2/f_{\text{eq}})$
	Power enhanced	90%	300%	$\mathcal{O}(\delta f_1/f_{\text{eq}})$
	NLL collisions	—	20%	$\mathcal{O}(\delta f_1/(f_{\text{eq}} \ln(1/g)))$
	Out-of-eq. particles	30%	400%	$\mathcal{O}(\delta f_1/f_{\text{eq}})$
Scalar EOM	Nucleation ( $T_n$ )	100%	—	$\mathcal{O}(g)$
	$V_{\text{eff}}$	—	20%	$\mathcal{O}(g)$

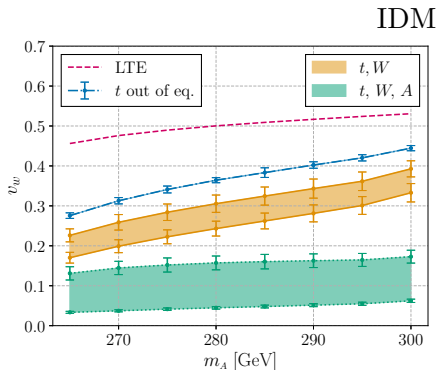
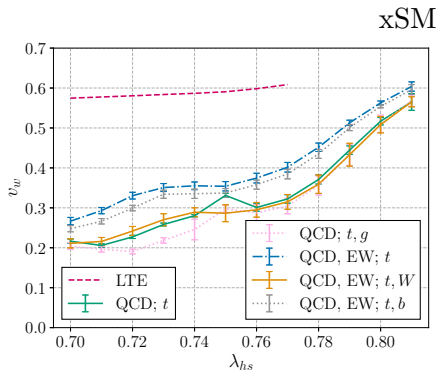
<sup>34</sup> van de Vis, Schicho, Niemi, Laurent, Hirvonen, Gould, *WallGo investigates: Theoretical uncertainties in the bubble wall velocity*, JHEP 04 (2026) 041 [2510.27691]

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**Improvement:**

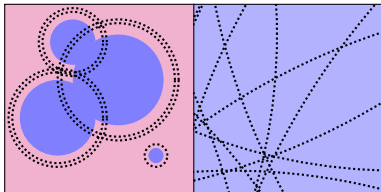
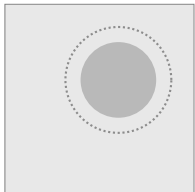
treat individual uncertainties or first-principle computations.<sup>35</sup>

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<sup>34</sup> van de Vis, Schicho, Niemi, Laurent, Hirvonen, Gould, *WallGo investigates: Theoretical uncertainties in the bubble wall velocity*, JHEP **04** (2026) 041 [2510.27691]

<sup>35</sup> Ai, Carosi, Garbrecht, Tamarit, Vanvlasselaer, *Bubble wall dynamics from nonequilibrium quantum field theory*, JHEP **08** (2025) 077 [2504.13725], Ramsey-Musolf, Zhu, *Bubble wall velocity from Kadanoff-Baym equations: fluid dynamics and microscopic interactions*, [2504.13724]

# Macroscopic scales



# Predicting the GW signal of colliding bubbles

Simulate numerically the scalar-fluid system consisting of

- ▷ Scalar field (gradient energy),
- ▷ Fluid motion.

For weak/intermediate phase transitions, the fluid motion is the dominant source of GWs.<sup>36</sup>

Simulated using phenomenological fluid-scalar interaction

$$\partial_{\mu} T_{\phi}^{\mu\nu} = -\eta_T u^{\mu} \partial_{\mu} \phi \partial^{\nu} \phi.$$

Recent computations of  $\eta_T(\phi)$  from first principles.<sup>37</sup>

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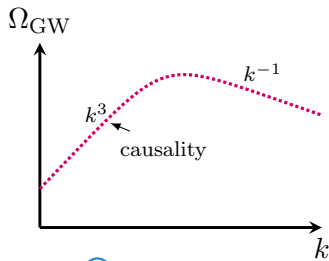
<sup>36</sup> Hindmarsh, Huber, Rummukainen, Weir, *Gravitational waves from the sound of a first order phase transition*, Phys. Rev. Lett. **112** (2014) 041301 [1304.2433]

<sup>37</sup> Ekstedt, Konstandin, van de Vis, *Scalar damping in cosmological phase transitions*, [2512.16663], Bhusal, Blasi, Konstandin, Perboni, van de Vis, *Slow-down of expanding bubbles in the early Universe*, [2603.22397]

# A conspiracy of bubbles: From single bubbles to a GW signal

Gradient energy of scalar field:

- ▷ Envelope approximation<sup>38</sup>



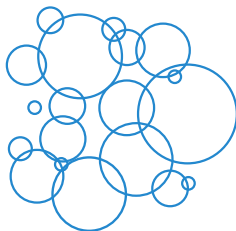
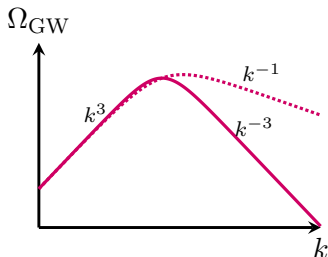
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<sup>38</sup> Kosowsky, Turner, *Gravitational radiation from colliding vacuum bubbles: envelope approximation to many bubble collisions*, Phys. Rev. D **47** (1993) 4372 [astro-ph/9211004], Weir, *Revisiting the envelope approximation: gravitational waves from bubble collisions*, Phys. Rev. D **93** (2016) 124037 [1604.08429]

# A conspiracy of bubbles: From single bubbles to a GW signal

Gradient energy of scalar field:

- ▷ Envelope approximation<sup>38</sup>
- ▷ Contributions of collided walls can change UV power law<sup>39</sup>



<sup>39</sup> Cutting, Hindmarsh, Weir, *Gravitational waves from vacuum first-order phase transitions: from the envelope to the lattice*, Phys. Rev. D **97** (2018) 123513 [1802.05712], Konstandin, *Gravitational radiation from a bulk flow model*, JCAP **03** (2018) 047 [1712.06869], Jinno, Takimoto, *Gravitational waves from bubble dynamics: Beyond the Envelope*, JCAP **01** (2019) 060 [1707.03111], Lewicki, Vaskonen, *Gravitational waves from colliding vacuum bubbles in gauge theories*, Eur. Phys. J. C **81** (2021) 437 [2012.07826], Gould, Sukuvaara, Weir, *Vacuum bubble collisions: From microphysics to gravitational waves*, Phys. Rev. D **104** (2021) 075039 [2107.05657]

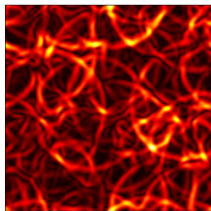
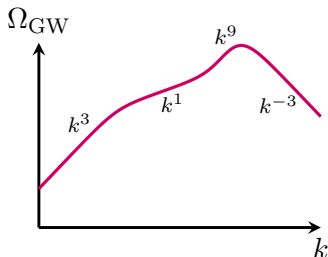
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Fluid motion:

Sound waves in fluid plasma<sup>40</sup>



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<sup>40</sup> Hindmarsh, Huber, Rummukainen, Weir, *Gravitational waves from the sound of a first order phase transition*, Phys. Rev. Lett. **112** (2014) 041301 [1304.2433], Hindmarsh, *Sound shell model for acoustic gravitational wave production at a first-order phase transition in the early Universe*, Phys. Rev. Lett. **120** (2018) 071301 [1608.04735], Sharma, Dahl, Brandenburg, Hindmarsh, *Shallow relic gravitational wave spectrum with acoustic peak*, [2308.12916], Jinno, Konstandin, Rubira, Stomberg, *Higgsless simulations of cosmological phase transitions and gravitational waves*, JCAP **02** (2023) 011 [2209.04369]

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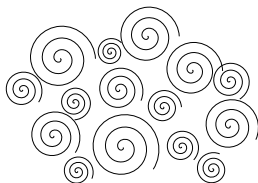
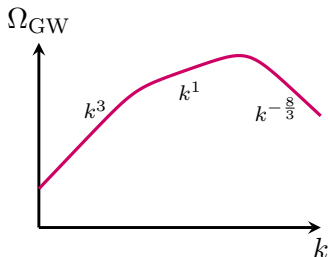
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Vortical turbulence: Kolmogorov<sup>41</sup>



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<sup>41</sup> Caprini, Durrer, Servant, *The stochastic gravitational wave background from turbulence and magnetic fields generated by a first-order phase transition*, JCAP **12** (2009) 024 [0909.0622], Roper Pol, Mandal, Brandenburg, Kahnishvili, Kosowsky, *Numerical simulations of gravitational waves from early-universe turbulence*, Phys. Rev. D **102** (2020) 083512 [1903.08585], Auclair, Caprini, Cutting et al., *Generation of gravitational waves from freely decaying turbulence*, JCAP **09** (2022) 029 [2205.02588], Dahl, Hindmarsh, Rummukainen, Weir, *Primordial acoustic turbulence: Three-dimensional simulations and gravitational wave predictions*, Phys. Rev. D **110** (2024) 103512 [2407.05826]

# A conspiracy of bubbles: From single bubbles to a GW signal

Gradient energy of scalar field:

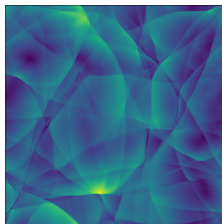
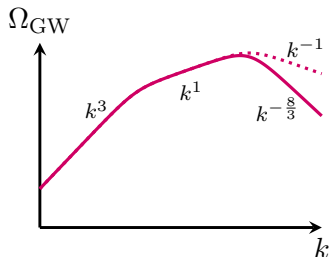
- ▷ Envelope approximation<sup>38</sup>
- ▷ Contributions of collided walls can change UV power law<sup>39</sup>

Fluid motion:

Sound waves in fluid plasma<sup>40</sup>

Vortical turbulence: Kolmogorov<sup>41</sup>

Acoustic turbulence: fluid shocks<sup>42</sup>



<sup>42</sup> Dahl, Hindmarsh, Rummukainen, Weir, *Decay of acoustic turbulence in two dimensions and implications for cosmological gravitational waves*, Phys. Rev. D **106** (2022) 063511 [2112.12013]

# A conspiracy of bubbles: From single bubbles to a GW signal

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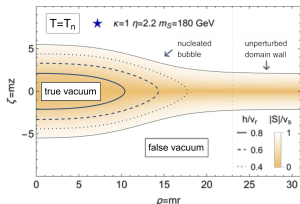
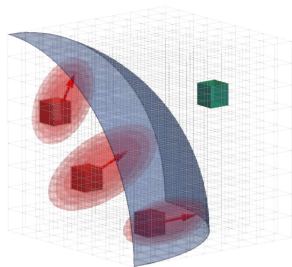
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Feebly interacting particles (FIP)<sup>43</sup>

Domain walls<sup>44</sup>

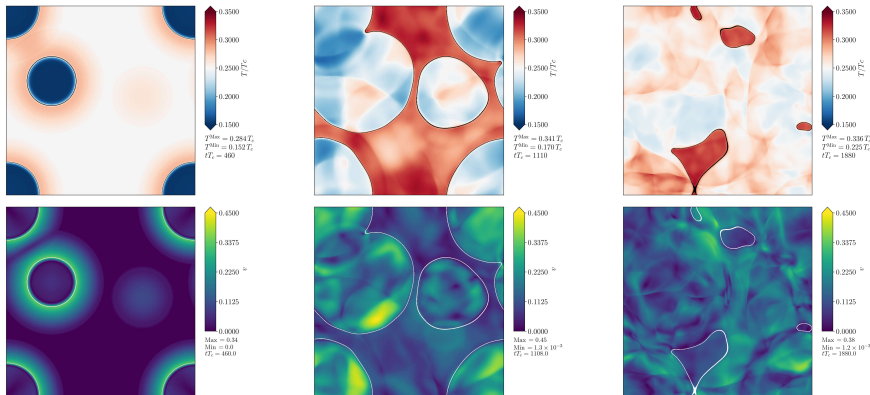


<sup>43</sup> Jinno, Shakya, van de Vis, *Gravitational Waves from Feebly Interacting Particles in a First Order Phase Transition*, Phys. Rev. Lett. **136** (2026) 131002 [2211.06405]

<sup>44</sup> Blasi, Mariotti, *Domain Walls Seeding the Electroweak Phase Transition*, Phys. Rev. Lett. **129** (2022) 261303 [2203.16450], Blasi, Jinno, Konstandin, Rubira, Stomberg, *Gravitational waves from defect-driven phase transitions: domain walls*, JCAP **10** (2023) 051 [2302.06952], Agrawal, Blasi, Mariotti, Nee, *Electroweak phase transition with a double well done doubly well*, JHEP **06** (2024) 089 [2312.06749]

# Slow-down of bubbles before collisions<sup>45</sup>

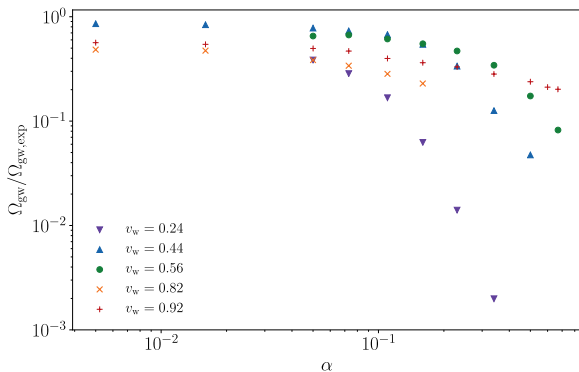
challenges the picture that the signal is completely determined by the microscopic scales. More advanced modelling of the signal is necessary.



<sup>45</sup> Cutting, Hindmarsh, Weir, *Vorticity, kinetic energy, and suppressed gravitational wave production in strong first order phase transitions*, Phys. Rev. Lett. **125** (2020) 021302 [1906.00480]

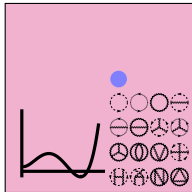
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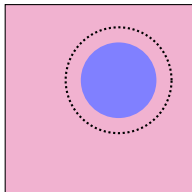
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# Conclusions: Progress and future challenges



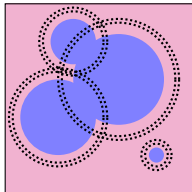
*Precision thermodynamics of BSM theories:*

- ▷ practical approach: **Effective Theories**
- ▷ Nucleation is hard. Discrepancy with lattice is becoming better.



*Wall velocity and hydrodynamics of single bubbles:*

- ▷ Can be predictive for the gravitational wave signal of colliding bubbles.



*Collective effects:*

- ▷ Fluid motion dominant source of GW signal,
- ▷ Simulations: decay of source and slow-down of bubbles, call for better modelling.

# The software landscape for PT induced GWs

